### ADVANCED PLACEMENT PHYSICS C TABLE OF INFORMATION

#### CONSTANTS AND CONVERSION FACTORS

<table>
<thead>
<tr>
<th>Physical Quantity</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proton mass, $m_p$</td>
<td>$1.67 \times 10^{-27}$ kg</td>
</tr>
<tr>
<td>Neutron mass, $m_n$</td>
<td>$1.67 \times 10^{-27}$ kg</td>
</tr>
<tr>
<td>Electron mass, $m_e$</td>
<td>$9.11 \times 10^{-31}$ kg</td>
</tr>
<tr>
<td>Avogadro’s number, $N_0$</td>
<td>$6.02 \times 10^{23}$ mol$^{-1}$</td>
</tr>
<tr>
<td>Universal gas constant, $R$</td>
<td>$8.31$ J/(mol-K)</td>
</tr>
<tr>
<td>Boltzmann’s constant, $k_B$</td>
<td>$1.38 \times 10^{-23}$ J/K</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Conversion Factor</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electron charge magnitude, $e$</td>
<td>$1.60 \times 10^{-19}$ C</td>
</tr>
<tr>
<td>1 electron volt, $1$ eV</td>
<td>$1.60 \times 10^{-19}$ J</td>
</tr>
<tr>
<td>Speed of light, $c$</td>
<td>$3.00 \times 10^{8}$ m/s</td>
</tr>
<tr>
<td>Universal gravitational constant, $G$</td>
<td>$6.67 \times 10^{-11}$ (N·m$^2$)/kg$^2$</td>
</tr>
<tr>
<td>Acceleration due to gravity at Earth’s surface, $g$</td>
<td>$9.8$ m/s$^2$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Physical Quantity</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1$ unified atomic mass unit, $1$ u</td>
<td>$1.66 \times 10^{-27}$ kg = $931$ MeV/$c^2$</td>
</tr>
<tr>
<td>Planck’s constant, $h$</td>
<td>$6.63 \times 10^{-34}$ J·s = $4.14 \times 10^{-15}$ eV·s</td>
</tr>
<tr>
<td>$hc$</td>
<td>$1.99 \times 10^{-25}$ J·m = $1.24 \times 10^{-3}$ eV·nm</td>
</tr>
<tr>
<td>Vacuum permittivity, $\varepsilon_0$</td>
<td>$8.85 \times 10^{-12}$ C$^2$/(N·m$^2$)</td>
</tr>
<tr>
<td>Coulomb’s law constant, $k$</td>
<td>$1/(4\pi\varepsilon_0) = 9.0 \times 10^9$ (N·m$^2$)/C$^2$</td>
</tr>
<tr>
<td>Vacuum permeability, $\mu_0$</td>
<td>$4\pi \times 10^{-7}$ (T·m)/A</td>
</tr>
<tr>
<td>Magnetic constant, $k’$</td>
<td>$\mu_0/(4\pi) = 1 \times 10^{-7}$ (T·m)/A</td>
</tr>
<tr>
<td>$1$ atmosphere pressure, $1$ atm</td>
<td>$1.0 \times 10^5$ N/m$^2$ = $1.0 \times 10^5$ Pa</td>
</tr>
</tbody>
</table>

#### UNIT SYMBOLS

<table>
<thead>
<tr>
<th>Physical Quantity</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>meter, m</td>
<td>m</td>
</tr>
<tr>
<td>kilogram, kg</td>
<td>kg</td>
</tr>
<tr>
<td>second, s</td>
<td>s</td>
</tr>
<tr>
<td>ampere, A</td>
<td>A</td>
</tr>
<tr>
<td>kelvin, K</td>
<td>K</td>
</tr>
<tr>
<td>hertz, Hz</td>
<td>Hz</td>
</tr>
<tr>
<td>newton, N</td>
<td>N</td>
</tr>
<tr>
<td>pascal, Pa</td>
<td>Pa</td>
</tr>
<tr>
<td>coulomb, C</td>
<td>C</td>
</tr>
<tr>
<td>ohm, Ω</td>
<td>Ω</td>
</tr>
<tr>
<td>farad, F</td>
<td>F</td>
</tr>
<tr>
<td>tesla, T</td>
<td>T</td>
</tr>
<tr>
<td>volt, V</td>
<td>V</td>
</tr>
<tr>
<td>degree Celsius, °C</td>
<td>°C</td>
</tr>
<tr>
<td>electron volt, eV</td>
<td>eV</td>
</tr>
<tr>
<td>joule, J</td>
<td>J</td>
</tr>
<tr>
<td>henry, H</td>
<td>H</td>
</tr>
</tbody>
</table>

#### PREFIXES

<table>
<thead>
<tr>
<th>Factor</th>
<th>Prefix</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^9$</td>
<td>giga</td>
<td>G</td>
</tr>
<tr>
<td>$10^6$</td>
<td>mega</td>
<td>M</td>
</tr>
<tr>
<td>$10^3$</td>
<td>kilo</td>
<td>k</td>
</tr>
<tr>
<td>$10^{-2}$</td>
<td>centi</td>
<td>c</td>
</tr>
<tr>
<td>$10^{-3}$</td>
<td>milli</td>
<td>m</td>
</tr>
<tr>
<td>$10^{-6}$</td>
<td>micro</td>
<td>μ</td>
</tr>
<tr>
<td>$10^{-9}$</td>
<td>nano</td>
<td>n</td>
</tr>
<tr>
<td>$10^{-12}$</td>
<td>pico</td>
<td>p</td>
</tr>
</tbody>
</table>

#### VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES

<table>
<thead>
<tr>
<th>Angle (θ)</th>
<th>$0^\circ$</th>
<th>$30^\circ$</th>
<th>$37^\circ$</th>
<th>$45^\circ$</th>
<th>$53^\circ$</th>
<th>$60^\circ$</th>
<th>$90^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>sinθ</td>
<td>0</td>
<td>$1/2$</td>
<td>$3/5$</td>
<td>$\sqrt{2}/2$</td>
<td>$4/5$</td>
<td>$\sqrt{3}/2$</td>
<td>1</td>
</tr>
<tr>
<td>cosθ</td>
<td>1</td>
<td>$\sqrt{3}/2$</td>
<td>$4/5$</td>
<td>$\sqrt{2}/2$</td>
<td>$3/5$</td>
<td>$1/2$</td>
<td>0</td>
</tr>
<tr>
<td>tanθ</td>
<td>0</td>
<td>$\sqrt{3}/3$</td>
<td>$3/4$</td>
<td>1</td>
<td>$4/3$</td>
<td>$\sqrt{3}$</td>
<td>$\infty$</td>
</tr>
</tbody>
</table>

The following assumptions are used in this exam.

I. The frame of reference of any problem is inertial unless otherwise stated.
II. The direction of current is the direction in which positive charges would drift.
III. The electric potential is zero at an infinite distance from an isolated point charge.
IV. All batteries and meters are ideal unless otherwise stated.
V. Edge effects for the electric field of a parallel plate capacitor are negligible unless otherwise stated.
### ADVANCED PLACEMENT PHYSICS C EQUATIONS

#### MECHANICS

- $v_x = v_{x0} + a_x t$
- $x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$
- $v_x^2 = v_{x0}^2 + 2a_x (x - x_0)$
- $\ddot{a} = \sum \frac{F}{m} = \ddot{F}_{net} / m$
- $\ddot{F} = \ddot{p} / m$
- $\ddot{J} = \int \ddot{F} \cdot d\ddot{r} = \Delta \ddot{p}$
- $|\ddot{F}_f| \leq \mu |\ddot{F}_N|
- $\Delta E = W = \int \ddot{F} \cdot d\ddot{r}$
- $K = \frac{1}{2} mv^2$
- $P = \frac{dE}{dt}$
- $P = \ddot{F} \cdot \ddot{v}$
- $\Delta U_g = mg\Delta h$
- $a_c = \frac{\ddot{v}^2}{r} = \omega^2 r$
- $\ddot{r} = \ddot{r} \times \ddot{F}$
- $\ddot{a} = \sum \ddot{a} / I = \ddot{a}_{net} / I$
- $I = \int \rho^2 dm = \sum mr^2$
- $\dddot{x} = \sum \dddot{x}_i / m = \dddot{x}_{net} / I$
- $P = \frac{dE}{dt}$
- $U_s = \frac{1}{2} k (\Delta x)^2$
- $T = \frac{2\pi}{\omega} = \int \frac{1}{f}$
- $T_s = 2\pi \sqrt{\frac{m}{k}}$
- $T_p = 2\pi \sqrt{\frac{E}{g}}$
- $\dddot{x} = \dddot{x}_{max} \cos(\omega t + \phi)$
- $r = r \omega$
- $\dddot{L} = \dddot{r} \times \dddot{p} = I \dddot{\omega}$
- $K = \frac{1}{2} I \omega^2$
- $\omega = \omega_0 + \alpha t$
- $\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$

#### ELECTRICITY AND MAGNETISM

- $[\dddot{F}_E] = \frac{1}{4\pi\epsilon_0} \left| q_1 q_2 \right| / r^2$
- $E = \dddot{F}_E / q$
- $\oint \dddot{E} \cdot d\dddot{A} = \frac{Q}{\epsilon_0}$
- $E_x = -\frac{dV}{dx}$
- $\Delta V = -\int \dddot{E} \cdot d\dddot{r}$
- $V = \frac{1}{4\pi\epsilon_0} \sum \frac{q_i}{r_i}$
- $U_E = qV = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$
- $\frac{1}{C_s} = \sum \frac{1}{C_i}$
- $I = \frac{dQ}{dt}$
- $U_C = \frac{1}{2} Q\Delta V = \frac{1}{2} C(\Delta V)^2$
- $d\dddot{B} = \frac{\mu_0 I}{4\pi} \frac{d\dddot{r} \times \dddot{r}}{r^2}$
- $R = \frac{\rho l}{A}$
- $E = \dddot{E} \cdot d\dddot{l} = -\frac{d\Phi_B}{dt}$
- $I = \frac{\Delta V}{R}$
- $I = Nev_d A$
- $I = \int B \cdot dA$
- $\Phi_B = \int B \cdot dA$
- $I = \frac{\Delta V}{R}$
- $R_s = \sum R_i$
- $U_L = \frac{1}{2} LI^2$
- $P = I(\Delta V)$
## GEOMETRY AND TRIGONOMETRY

**Rectangle**
\[ A = bh \]

**Triangle**
\[ A = \frac{1}{2}bh \]

**Circle**
\[ A = \pi r^2 \]
\[ C = 2\pi r \]
\[ s = r\theta \]

**Rectangular Solid**
\[ V = \ell wh \]

**Cylinder**
\[ V = \pi r^2\ell \]
\[ S = 2\pi r\ell + 2\pi r^2 \]

**Sphere**
\[ V = \frac{4}{3}\pi r^3 \]
\[ S = 4\pi r^2 \]

**Right Triangle**
\[ a^2 + b^2 = c^2 \]
\[ \sin \theta = \frac{a}{c} \]
\[ \cos \theta = \frac{b}{c} \]
\[ \tan \theta = \frac{a}{b} \]

## CALCULUS

\[ \frac{df}{dx} = \frac{df}{du} \frac{du}{dx} \]
\[ \frac{d}{dx} (x^n) = nx^{n-1} \]
\[ \frac{d}{dx} (e^{ax}) = ae^{ax} \]
\[ \frac{d}{dx} (\ln ax) = \frac{1}{x} \]
\[ \frac{d}{dx} [\sin (ax)] = a \cos (ax) \]
\[ \frac{d}{dx} [\cos (ax)] = -a \sin (ax) \]
\[ \int x^n dx = \frac{1}{n+1} \cdot x^{n+1} , n \neq -1 \]
\[ \int e^{ax} dx = \frac{1}{a} e^{ax} \]
\[ \int \frac{dx}{x + a} = \ln |x + a| \]
\[ \int \cos (ax) dx = \frac{1}{a} \sin (ax) \]
\[ \int \sin (ax) dx = -\frac{1}{a} \cos (ax) \]

## VECTOR PRODUCTS
\[ \vec{A} \cdot \vec{B} = AB \cos \theta \]
\[ |\vec{A} \times \vec{B}| = AB \sin \theta \]
1. The circuit represented above is composed of three resistors with the resistances shown, a battery of voltage $V_0$, a capacitor of capacitance $C$, and a switch $S$. The switch is closed, and after a long time, the circuit reaches steady-state conditions. Answer the following questions in terms of $V_0$, $R$, $C$, and fundamental constants, as appropriate.

   (a) Derive an expression for the steady-state current supplied by the battery.
   
   (b) Derive an expression for the charge on the capacitor.
   
   (c) Derive an expression for the energy stored in the capacitor.

Now the switch is opened at time $t = 0$.

   (d) Write, but do NOT solve, a differential equation that could be used to solve for the charge $q(t)$ on the capacitor as a function of the time $t$ after the switch is opened.
2019 AP® PHYSICS C: ELECTRICITY AND MAGNETISM FREE-RESPONSE QUESTIONS

(e)

i. Calculate the current in resistor R immediately after the switch is opened.

ii. On the axes below, sketch the current in the circuit as a function of time from time $t = 0$ to a long time after the switch is opened. Explicitly label the maxima with numerical values or algebraic expressions, as appropriate.

![Current vs Time Graph]

(f) Is the total amount of energy dissipated in the resistors after the switch is opened greater than, less than, or equal to the amount of energy stored in the capacitor calculated in part (e)?

_____ Greater than  _____ Less than  _____ Equal to

Justify your answer.
2. A nonconducting hollow sphere of inner radius 0.030 m and outer radius 0.050 m carries a positive volume charge density \( \rho \), as shown in the figure above. The charge density \( \rho \) of the sphere is given as a function of the distance \( r \) from the center of the sphere, in meters, by the following.

\[
\begin{align*}
& \text{\( r < 0.030 \text{ m} \): } \rho = 0 \\
& \text{\( 0.030 \text{ m} < r < 0.050 \text{ m} \): } \rho = b/r, \text{ where } b = 1.6 \times 10^{-6} \text{ C/m}^2 \\
& \text{\( r > 0.050 \text{ m} \): } \rho = 0
\end{align*}
\]

(a) Calculate the total charge of the sphere.

(b) Using Gauss’s law, calculate the magnitude of the electric field \( E \) at the outer surface of the sphere.

(c) On the axes below, sketch the magnitude of the electric field \( E \) as a function of distance \( r \) from the center of the sphere.
(d) Calculate the electric potential $V$ at the outer surface of the sphere. Assume the electric potential to be zero at infinity.

(e) A proton is released from rest at the outer surface of the sphere at time $t = 0 \text{ s}$.
   i. Calculate the magnitude of the initial acceleration of the proton.
   ii. Calculate the speed of the proton after a long time.
3. Two plates are set up with a potential difference $V$ between them. A small sphere of mass $m$ and charge $-e$ is placed at the left-hand plate, which has a negative charge, and is allowed to accelerate across the space between the plates and pass through a small opening. After passing through the small opening, the sphere enters a region in which there is a uniform magnetic field of magnitude $B$ directed into the page, as shown above. Ignore gravitational effects. Express all algebraic answers in terms of $V$, $m$, $e$, $B$, and fundamental constants, as appropriate.

(a)

i. What is the initial direction of the force on the sphere as it enters the magnetic field?

- ____ Into the page
- ____ Out of the page
- ____ Toward the top of the page
- ____ Toward the bottom of the page

ii. Describe the path taken by the sphere after it enters the magnetic field.

(b) Derive an expression for the speed of the sphere as it passes through the small opening.

(c) Derive an expression for the radius of the path taken by the sphere as it moves through the magnetic field.
An experiment is performed in which a beam of electrons is accelerated across the space between the plates and passes through the small opening. After passing through the opening, the electrons travel in a semicircular path and strike the right-hand plate. The potential difference between the plates is varied in regular increments, as shown in the table below. For each potential difference, the magnetic field is varied in order to cause the beam to strike the right-hand plate at a distance of 0.020 m from the opening.

<table>
<thead>
<tr>
<th>Potential difference (V)</th>
<th>60</th>
<th>70</th>
<th>100</th>
<th>110</th>
<th>120</th>
<th>140</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnetic field (T × 10^{-3})</td>
<td>2.62</td>
<td>2.78</td>
<td>3.39</td>
<td>3.54</td>
<td>3.78</td>
<td>3.99</td>
</tr>
</tbody>
</table>

(d) Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the mass-to-charge ratio of an electron.

Vertical axis: ____________
Horizontal axis: ____________

Use the remaining columns in the table above, as needed, to record any quantities that you indicated that are not given. Label each column you use and include units.

(e) On the graph below, plot the relationship determined in part (d). Clearly scale and label all axes, including units, if appropriate. Draw a straight line that best represents the data.

(f) Using the straight line from part (e), determine the mass-to-charge ratio of an electron.