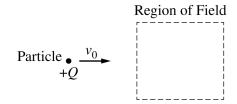
AP Physics 2: Algebra-Based Scoring Guidelines

General Notes About 2019 AP Physics Scoring Guidelines

- 1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.
- 2. The requirements that have been established for the paragraph-length response in Physics 1 and Physics 2 can be found on AP Central at https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf.
- 3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.
- 4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections Student Presentation" in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or "Terms Defined" in the *AP Physics 1: Algebra-Based Course and Exam Description* and the *AP Physics 2: Algebra-Based Course and Exam Description*.
- 5. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but the use of 10 m/s^2 is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.
- 6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

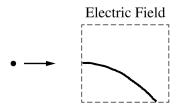
Question 1

10 points



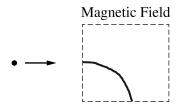
The figure above shows a particle with positive charge +Q traveling with a constant speed v_0 to the right and in the plane of the page. The particle is approaching a region, shown by the dashed box, that contains a constant uniform field. The effects of gravity are negligible.

On the figure below, draw a possible path of the particle in the region if the region contains only an electric field directed toward the bottom of the page.



For a curved path that is initially horizontal and does not have a component of velocity toward the left	1 point
For a path that deflects toward the bottom of the page and reaches an edge of the region	1 point

On the figure below, draw a possible path of the particle in the region if the region contains only a magnetic field directed out of the page.



For a curved path that is initially horizontal, is not more than a semicircle, and reaches	1 point
an edge of the region	
For a path that deflects toward the bottom of the page	1 point

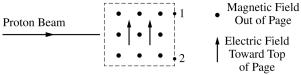
Question 1 (continued)

- (a) (continued)
 - iii. LO 2.C.5.3, SP 1.1, 7.1 1 point

For which of the previous situations is the motion more similar to that of a projectile in only a gravitational field near Earth's surface, and why?

For indicating that the motion in the electric field is more similar to a projectile because	1 point
the force or acceleration is always down or constant, or the shape is parabolic	

(b) LO 2.D.1.1, SP 2.2; LO 3.A.3.4, SP 6.1, 6.4; LO 3.B.1.4, SP 6.4, 7.2; LO 3.B.2.1, SP 1.1, 1.4, 2.2 5 points



Another region of space contains an electric field directed toward the top of the page and a magnetic field directed out of the page. Both fields are constant and uniform. A horizontal beam of protons with a variety of speeds enters the region, as shown above. Protons exit the region at a variety of locations, including points 1 and 2 shown on the figure. In a coherent, paragraph-length response, explain why some protons exit the region at point 1 and others exit at point 2. Use physics principles to explain your reasoning.

Equip directing that initially the alactic and magnetic forces act in apparite directions	1
For indicating that initially the electric and magnetic forces act in opposite directions	1 point
For indicating or implying that the magnetic force is affected by speed, but the electric	1 point
force is not	
For indicating that different paths occur as a result of the addition of forces	1 point
For indicating that slower protons exit higher than faster protons (i.e., slower protons	1 point
exit at point 1 and faster protons exit at point 2)	•
For a logical, relevant, and internally consistent argument that addresses the question	1 point
asked and follows the guidelines described in the published requirements for the	
paragraph-length response	
Example:	
For a charged particle to travel through the region undeflected, the net force on it must	
be zero. This means that the upward electric force and the downward magnetic force	
must be equal and opposite to each other. This occurs for a particular speed. The	
electric force is independent of the particle's velocity, but the magnetic force will be	
larger for greater velocities and less for smaller velocities. If a particle is moving	
faster than the particular speed, it will experience a greater magnetic force and be	
deflected downward. If it is moving more slowly than the particular speed, it will be	
deflected upward.	

Question 1 (continued)

(b) (continued)

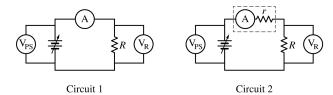
Claim: Slower protons exit higher than faster protons (i.e., slower protons exit at point 1	
and faster protons exit at point 2).	
Evidence: The electric and magnetic forces act in opposite directions. The magnetic	
force is affected by speed, but the electric force is not.	
Reasoning: Different paths occur as a result of the addition of forces.	

Learning Objectives

- **LO 2.C.1.1:** The student is able to predict the direction, and the magnitude of the force exerted on an object with an electric charge q placed in an electric field E using the mathematical model of the relation between an electric force and an electric field: $\vec{F} = q\vec{E}$; a vector relation. [See Science Practices 6.4, 7.2]
- **LO 2.C.5.3:** The student is able to represent the motion of an electrically-charged particle in the uniform field between two oppositely charged plates and express the connection of this motion to projectile motion of an object with mass in Earth's gravitational field. [See Science Practices 1.1, 2.2, 7.1]
- **LO 2.D.1.1:** The student is able to apply mathematical routines to express the force exerted on a moving charged object by a magnetic field. [See Science Practices 2.2]
- **LO 3.A.3.4:** The student is able to make claims about the force on an object due to the presence of other objects with the same property: mass, electric charge. [See Science Practices 6.1, 6.4]
- **LO 3.B.1.4:** The student is able to predict the motion of an object subject to forces exerted by several objects using an application of Newton's second law in a variety of physical situations. [See Science Practices 6.4, 7.2]
- **LO 3.B.2.1:** The student is able to create and use free-body diagrams to analyze physical situations to solve problems with motion qualitatively and quantitatively. [See Science Practices 1.1, 1.4, 2.2]
- **LO 3.C.3.1:** The student is able to use right-hand rules to analyze a situation involving a current-carrying conductor and a moving electrically charged object to determine the direction of the magnetic force exerted on the charged object due to the magnetic field created by the current-carrying conductor. [See Science Practices 1.4]

Question 2

12 points

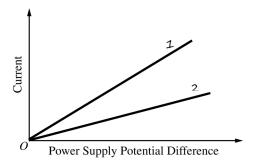


The two circuits shown above contain an ideal variable power supply, an ohmic resistor of resistance R, an ammeter A, and two voltmeters V_{PS} and V_{R} . In circuit 1 the ammeter has negligible resistance, and in circuit 2 the ammeter has significant internal ohmic resistance r. The potential difference of the power supply is varied, and measurements of current and potential difference are recorded.

(a) LO 4.E.5.1, SP 6.4 2 points

The axes below can be used to graph the current measured by the ammeter as a function of the potential difference measured across the power supply. On the axes, do the following.

- Sketch a possible graph for circuit 1 and label it 1.
- Sketch a possible graph for circuit 2 and label it 2.



For graph 1 a straight line with a positive slope through origin	1 point
For graph 2 a straight line with a positive slope through origin with a smaller slope than line 1	1 point

(b) LO 5.B.9.6, SP 2.2; LO 5.C.3.4, SP 6.4 2 points

Let ΔV_{PS} be the potential difference measured by voltmeter V_{PS} across the power supply, and let I be the current measured by the ammeter A. For each circuit, write an equation that satisfies conservation of energy, in terms of ΔV_{PS} , I, R, and r, as appropriate.

<u>Circuit 1</u> <u>Circuit 2</u>

For a correct equation for circuit 1	1 point
$\Delta V_{PS} - IR = 0$	
For a correct equation for circuit 2	1 point
$\Delta V_{PS} - I(R+r) = 0$	

Question 2 (continued)

(c) LO 5.B.9.8, SP 1.5 2 points

Explain how your equations in part (b) account for any differences between graphs 1 and 2 in part (a).

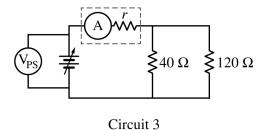
For indicating that the slope is inversely proportional to the resistance	1 point
For explaining that the equations in part (b) show that a larger total resistance	1 point
corresponds to a smaller slope or smaller current	
Example:	
Claim: The equations in part (b) account for the differences between graphs 1 and 2 in	
part (a).	
Evidence: The graphs show a linear relationship between current and potential	
difference. The equations are linear functions, which when graphed would have a	
slope that is the inverse of the total resistance.	
Reasoning: The difference between the equations is the value of the total resistance, so	
the equations account for the difference in slopes. The larger the total resistance, the	
smaller the slope.	

(d) LO 5.B.9.6, SP 2.2; LO 5.C.3.4, SP 6.4, 7.2 2 points

In circuit 2, $R = 40 \Omega$. When voltmeter V_{PS} reads 3.0 V, voltmeter V_{R} reads 2.5 V. Calculate the internal resistance r of the ammeter.

Ohm's law solution:	
For correctly calculating the current in the circuit	1 point
$I = \Delta V_R / R = 2.5 \text{ V} / 40 \Omega = 0.0625 \text{ A}$	
For using Ohm's law with the calculated current and correct potential difference	1 point
$r = \Delta V_r / I = (3 \text{ V} - 2.5 \text{ V}) / 0.0625 \text{ A}$	
$r = 8 \Omega$	

(e) $\mbox{Voltmeter } V_R \mbox{ in circuit 2 is replaced by a resistor with resistance } 120 \ \Omega \mbox{ to create circuit 3 shown below.} \\ \mbox{Voltmeter } V_{PS} \mbox{ still reads } 3.0 \ V.$



Question 2 (continued)

- (e) (continued)
 - i. LO 4.E.5.1, SP 2.2 2 points

Calculate the equivalent resistance R_{eq} of the circuit.

For calculating the equivalent resistance of the parallel branches		1 point
$\frac{1}{40\ \Omega} + \frac{1}{120\ \Omega} = \frac{4}{120\ \Omega}$		
$R_{\parallel} = 30 \ \Omega$		
For adding the value of r from part (d) to R_{\parallel}		1 point
$R_{eq} = 30 \Omega + 8 \Omega = 38 \Omega$		

ii. LO 5.B.9.6, SP 2.2 2 points

Calculate the current in each of the resistors that are in parallel.

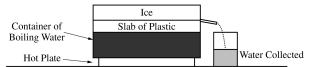
For substituting the correct potential difference and the resistance from part (e)(i) into Ohm's law to determine the current through the battery		
$I_{tot} = 3 \text{ V}/38 \Omega = 0.079 \text{ A}$		
For calculating two currents that are in the correct ratio ($I_{40 \Omega} = 3I_{120 \Omega}$)		1 point
$\Delta V_{\text{parallel}} = (3 \text{ V}) - (8 \Omega)(0.079 \text{ A}) = 2.36 \text{ V}$		
$I_{40 \Omega} = \frac{2.36 \text{ V}}{40 \Omega} = 0.059 \text{ A}$		
$I_{120 \Omega} = \frac{2.36 \text{ V}}{120 \Omega} = 0.020 \text{ A}$		

Learning Objectives

- **LO 4.E.5.1:** The student is able to make and justify a quantitative prediction of the effect of a change in values or arrangements of one or two circuit elements on the currents and potential differences in a circuit containing a small number of sources of emf, resistors, capacitors, and switches in series and/or parallel. [See Science Practices 2.2, 6.4]
- **LO 5.B.9.6:** The student is able to mathematically express the changes in electric potential energy of a loop in a multiloop electrical circuit and justify this expression using the principle of the conservation of energy. [See Science Practices 2.1, 2.2]
- **LO 5.B.9.8:** The student is able to translate between graphical and symbolic representations of experimental data describing relationships among power, current, and potential difference across a resistor. [See Science Practices 1.5]
- **LO 5.C.3.4:** The student is able to predict or describe current values in series and parallel arrangements of resistors and other branching circuits using Kirchhoff's junction rule and explain the relationship of the rule to the law of charge conservation. [See Science Practices 6.4, 7.2]

Question 3

12 points



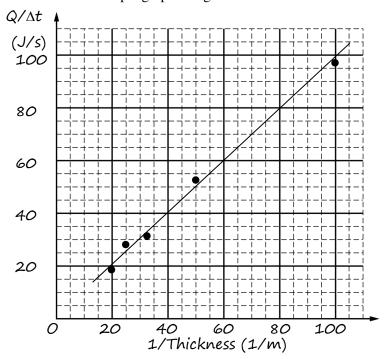
A group of students use the apparatus shown above to determine the thermal conductivity of a certain type of plastic. A hot plate is used to keep water in a container boiling at a temperature of 100°C . They place a slab of the plastic with area $0.025~\text{m}^2$ and thickness 0.010~m above the container so that the bottom surface of the slab is at a temperature of 100°C . They put a large block of ice with temperature 0°C on top of the plastic slab. Some of the ice melts, and the students measure the amount of water collected during a time Δt . The students correctly calculate the amount of energy Q delivered to the ice and thus determine $Q/\Delta t$. They repeat this experiment several times, each time adding an identical slab to increase the total thickness L of plastic. Their results are shown in the table below.

Table with sample entries for part (a)(ii)

Energy Flow Rate $Q/\Delta t$ (J/s)	97	53	31	27	18
Total Thickness of Plastic (m)	0.01	0.02	0.03	0.04	0.05
1/Thickness (1/m)	100	50	33.3	25	20

(a) The students want to create a graph to yield a straight line whose slope could be used to calculate the thermal conductivity of the plastic.

Sample graph using above data



Question 3 (continued)

- (a) (continued)
 - i. LO 1.E.3.1, SP 4.1, 5.1 1 point

Label the axes below to indicate a pair of quantities that could be graphed to yield a straight line. Include units for the quantities.

$\frac{Q}{\Delta t} = \frac{kA \Delta T}{L}$	
For labeling the axes with two quantities that would produce a linear graph, including units	1 point
Example: $Q/\Delta t$ and 1/thickness	

ii. LO 1.E.3.1, SP 4.1, 5.1 3 points

On the grid on the previous page, create a linear graph using the values for the quantities indicated in part (a)(i). Be sure to do the following:

- Add to the data table the values of any quantities to be plotted that are not already given.
- Scale the axes.
- Plot the data from the table.
- Draw a line that best represents the data.

For scaling the axes linearly so the data extends over at least half of each axis	1 point
For accurately plotting the data	1 point
For a best-fit curve or line that fits the trend in the data	1 point

iii. LO 1.E.3.1, 5.1 2 points

Use the graph to calculate the thermal conductivity of the plastic.

For a correct method for calculating the slope using points on the best-fit line	1 point
For the graph above, slope = $\frac{(80 - 20)(J/s)}{(80 - 20)(1/m)} = 1.0 \text{ J} \cdot \text{m/s}$	
For determining the thermal conductivity k , with or without units using the slope found	1 point
above	
$\frac{Q}{\Delta t} = \frac{kA \Delta T}{L} \text{ so slope} = kA \Delta T$	
Using slope above: $k = \text{slope}/A \Delta T = 1(J \cdot m/s)/(0.025 \text{ m}^2)(100^{\circ} \text{ C}) = 0.40 \text{ J/s} \cdot m^{\circ} \text{C}$	

Question 3 (continued)

(b) LO 5.B.6.1, SP1.2 2 points

Indicate one potential problem with the setup that could lead to an experimental value for the thermal conductivity that is different from the actual value. Use physics principles to explain the effect this problem could have on the experimental value.

For any valid indication of an additional thermal interaction with the environment	1 point
For a reasonable explanation of how additional energy added or lost could change the	1 point
experimental value of conductivity	
Example 1: The given setup allows energy to be transferred to the ice from the air	
around it. This means the values of $Q/\Delta t$ contain energy that did not go through the	
plastic slab, resulting in a value of k that is too large.	
Example 2: The given setup allows energy to be lost out the sides of the plastic slab.	
This means the values of $Q/\Delta t$ do not contain all the energy that went through the	
plastic slab, resulting in a value of k that is too small.	
Claim: The problem leads to a value of <i>k</i> that is too small/large.	
Evidence: The problem allows energy transfer into/out of the system that is not	
accounted for.	
Reasoning: The values of $Q/\Delta t$ contain less/more energy than went through the plastic	
slab, resulting in a value of k that is too small/large.	

(c)	LO 4.C.3.1, SP 6.4
	1 point

The rectangle below represents a side view of the plastic slab. Draw a single arrow on the diagram representing the direction of the net flow of energy through the plastic.

	<u> </u>	
For drawing an arrow toward the top o	of the page	1 point

(d) LO 4.C.3.1, SP 6.4; LO 5.B.6.1, SP 1.2; LO 5.D.1.6, SP 6.4 2 points

Describe what occurs in the plastic at the microscopic level that explains the energy flow you indicated in part (c).

For indicating that particles at the bottom (or a location consistent with part (c)) have a	1 point
higher temperature or kinetic energy, so they vibrate faster	
For indicating that particles collide with neighboring particles, transferring energy from	1 point
faster to slower particles in the process	
Example: Energy absorbed at the lower surface makes particles jiggle faster, they jiggle	
particles above them, and so forth until energy reaches the other side.	

Question 3 (continued)

(e) LO 1.E.3.1, SP 4.1, 4.2 1 point

An extra plastic slab sits on a wood surface, with both the plastic slab and the wood surface at room temperature. A student touches each and finds that the plastic slab feels cooler than the wood surface. Explain what causes this observation.

For indicating that the slab and wood have different thermal conductivities or that	1 point
energy is transferred into the plastic and wood at different rates, with no incorrect	
statements	

Learning Objectives

- **LO 1.E.3.1:** The student is able to design an experiment and analyze data from it to examine thermal conductivity. [See Science Practices 4.1, 4.2, 5.1]
- **LO 4.C.3.1:** The student is able to make predictions about the direction of energy transfer due to temperature differences based on interactions at the microscopic level. [See Science Practices 6.4]
- **LO 5.B.6.1:** The student is able to describe the models that represent processes by which energy can be transferred between a system and its environment because of differences in temperature: conduction, convection, and radiation. [See Science Practices 1.2]
- **LO 5.D.1.6:** The student is able to make predictions of the dynamical properties of a system undergoing a collision by application of the principle of linear momentum conservation and the principle of the conservation of energy in situations in which an elastic collision may also be assumed. [See Science Practices 6.4]

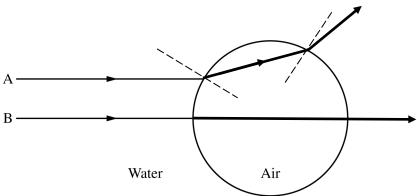
Question 4

10 points

A student notices many air bubbles rising through the water in a large fish tank at an aquarium.

(a) LO 6.E.3.1, SP 1.1, 1.4 3 points

In the figure below, the circle represents one such air bubble, and two incoming rays of light, A and B, are shown. Ray B points toward the center of the circle. On the diagram, draw the paths of rays A and B as they go through the bubble and back into the water. Your diagram should clearly show what happens to the rays at each interface.



For ray B going straight through		1 point
For ray A bending away from the normal as it enters the air from the water		1 point
For ray A bending the opposite direction in relationship to the normal as it exits the air		1 point
and enters the water compared to the refraction entering the air from the water	1	_
Note: The normals need not be shown.		

(b) LO 5.B.4.1, SP 6.4, 7.2; LO 5.B.4.2, SP 1.4, 7.2; LO 5.B.5.4, SP 6.4, 2.2; LO 5.B.5.5, SP 2.2, 6.4 3 points

The bubble has a volume V_1 , the air inside it has density ρ_A , and the water around it has density ρ_W . The bubble starts at rest and has a speed v_f when it has risen a height h. Assume that the change in the bubble's volume is negligible. Derive an expression for the mechanical energy dissipated by drag forces as the bubble rises this distance. Express your answer in terms of the given quantities and fundamental constants, as appropriate.

For a valid application of the work-energy theorem	1 point
$\Delta K = W_{\text{net}} = W_b - W_g - W_{\text{diss}} $	
For finding the work done by the buoyant force	1 point
$W_b = \rho_w V_1 g h$	
For correct substitutions into an equation with consistent relative signs for the terms	1 point
$\frac{1}{2}\rho_A V_1 v_f^2 = \rho_W V_1 g h - \rho_A V_1 g h - W_{\text{diss}} $	
$ W_{\text{diss}} = \rho_W V_1 g h - \rho_A V_1 g h - \frac{1}{2} \rho_A V_1 v_f^2$	

Question 4 (continued)

(c)

At a particular instant, one bubble is 4.5 m below the water's surface. The surface of the water is at sea level, and the density of the water is 1000 kg/m^3 .

i. LO 5.B.10.1, SP 2.2 1 point

Determine the absolute pressure in the bubble at this location.

$P_{4.5\text{m}} = P_{atm} + \rho_w g d$	
$P_{4.5\text{m}} = (1.0 \times 10^5 \text{Pa}) + (1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(4.5 \text{ m})$	
For a correct answer with units	1 point
$P_{4.5\text{m}} = 1.44 \times 10^5 \text{ Pa (or } 1.45 \times 10^5 \text{ Pa using } g = 10 \text{ m/s}^2 \text{)}$	

ii. LO 7.A.3.3, SP 5.1 2 points

The bubble has a volume V_1 when it is 4.5 m below the water's surface. Assume that the temperature of the air in the bubble remains constant as it rises. In terms of V_1 , calculate the volume of the bubble when it is just below the surface of the water.

For applying the ideal gas law at two locations in an attempt to determine the new bubble volume	1 point
$P_{4.5\text{m}}V_1 = P_{atm}V_{surface}$	
$V_{surface} = P_{4.5m}V_1/P_{atm}$	
For substituting pressures consistent with part (i)	1 point
$V_{surface} = \left(1.44 \times 10^5 \text{ Pa}\right) V_1 / \left(1 \times 10^5 \text{ Pa}\right)$	
$V_{surface} = 1.44V_1 \text{ (or } 1.45V_1 \text{ using } g = 10 \text{ m/s}^2 \text{)}$	

iii. LO 7.A.3.3, SP 5.1 1 point

If the air in the bubble cooled as it rose, the volume of the bubble would be less than the value calculated in part (c)(ii). Use physics principles to briefly explain why.

For a correct explanation	1 point
Note: The explanation may be qualitative or quantitative. The explanation may also be	
macroscopic or microscopic.	

Question 4 (continued)

(c) (continued) iii. (continued)

Example 1: By the ideal gas law, $P_{4.5\text{m}}V_1/T_1 = P_{atm}V_{surface}/T_{surface}$, so	
$V_{surface} = P_{4.5 \text{m}} V_1 T_{surface} / P_{atm} T_1$. The two pressures still have their previous values.	
$T_{surface} < T_1$, so the volume at the surface will be smaller.	
Example 2: As the bubble cools, the air molecules move slower. Slower molecules exert less force on the inner surface of the bubble. The unbalanced force, due to the difference in the forces on the inside and outside of the bubble, causes the bubble to expand less than it did in the constant temperature situation or contract.	
Claim (given): The volume of the bubble will decrease	
Example 1 evidence: $P_{4.5\text{m}}V_1/T_1 = P_{atm}V_{surface}/T_{surface}, \text{ so } V_{surface} = P_{4.5\text{m}}V_1T_{surface}/P_{atm}T_1$	
Example 1 reasoning: The two pressures still have their previous values. $T_{surface} < T_1$, so the volume at the surface will be smaller.	
Example 2 evidence: As the bubble cools, the air molecules move slower. Slower molecules exert less force on the inner surface of the bubble. Example 2 reasoning: The unbalanced force, due to the difference in the forces on the inside and outside of the bubble, causes the bubble to contract.	

Learning Objectives:

- **LO 5.B.4.1:** The student is able to describe and make predictions about the internal energy of systems. [See Science Practices 6.4, 7.2]
- **LO 5.B.4.2:** The student is able to calculate changes in kinetic energy and potential energy of a system using information from representations of that system. [See Science Practices 1.4, 2.1, 2.2]
- **LO 5.B.5.4:** The student is able to make claims about the interaction between a system and its environment in which the environment exerts a force on the system, thus doing work on the system and changing the energy of the system (kinetic energy plus potential energy). [See Science Practices 6.4, 7.2]
- **LO 5.B.5.5:** The student is able to predict and calculate the energy transfer to (i.e., the work done on) an object or system from information about a force exerted on the object or system through a distance. [See Science Practices 2.2, 6.4]
- **LO 5.B.10.1:** The student is able to make calculations related to a moving fluid using Bernoulli's equation. [See Science Practices 2.2]
- **LO 6.E.3.1:** The student is able to describe models of light traveling across a boundary from one transparent material to another when the speed of propagation changes, causing a change in the path of the light ray at the boundary of the two media. [See Science Practices 1.1, 1.4]
- **LO 7.A.3.3:** The student is able to analyze graphical representations of macroscopic variables for an ideal gas to determine the relationships between these variables and to ultimately determine the ideal gas law PV = nRT. [See Science Practices 5.1]